Optimal Systemic Risk Mitigation in Financial Networks

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Outline

Introduction

Financial network

- Institutions are connected via bilateral exposures originating from trades
- The intricate structure of linkages captured via network representation of financial system

Systemic risk

- The risk of collapse of entire financial network
- Triggered by shock transmission mechanism generated by recursive interdependence in the network

see also Allen & Gale (2001); Eisenberg & Noe (2001); Gai & Kapadia (2010)

Our Goal

Construct a framework which allows

- capturing systemic consequences of the default of a node on the state of the entire network
- mitigating systemic risk through optimal bailout strategies in the form of loans
- providing a tool to support regulator decisions of when/how to intervene to mitigate systemic risk

Financial Network I

- Modeled as a digraph where the set V of nodes represents financial firms, and edges the liability relationships between nodes
- Multi-period model with finite time horizon T. Each period is a discrete interval $[t, t+1)$, $t \in \{0, 1, \ldots, T-1\}$

State of network in period $[t, t+1)$ characterized by

- $\mathbf{L}^t$: matrix of interbank liabilities due at t, with L_{ij}^t denoting liabilities that i owes to j at t
- ι^t: vector of operating cash inflows at t
- $\mathbf{a^t}$: vector of illiquid assets owned by nodes at t

Financial Network II

- $l_i^t := \sum_{j \neq i, j \in V^t} L_{ij}^t$: total liabilities node i owes to other nodes at t
- $\Pi_{ij}^t := \frac{L_{ij}^t}{l_i^t} \mathbf{1}_{l_i^t > 0}$: fraction of nodes i's total liabilities owed to j at t
- p_i^t: payment node i makes to other nodes at t
- v_i^t: cash available to node i at t
- $c_i^t := \sum_{j \neq i, i \in V^t} \Pi_{ji} p_j^t + \iota_i^t + v_i^t$

Bailout Rules

- Lender of Last Resort (LLR) can provide bailout loans to illiquid but solvent nodes so to mitigate financial distress caused by default
- Each node in network uses cash left after paying due liabilities to his creditors, to repay currently owed bailout amount.
- Define the bailout quantities
  - o_i^t: bailout loan granted by LLR to rescue node i at t
  - b_i^t: amount that node i uses to repay his outstanding bailout loan at t.
  - q_i^t: portion of bailout loan node i still needs to repay at t, i.e. $\mathbf{q}^{t+1} = (1 + r_c)(\mathbf{q}^t + \mathbf{o}^t - \mathbf{b}^t)$.

Default Modeling I

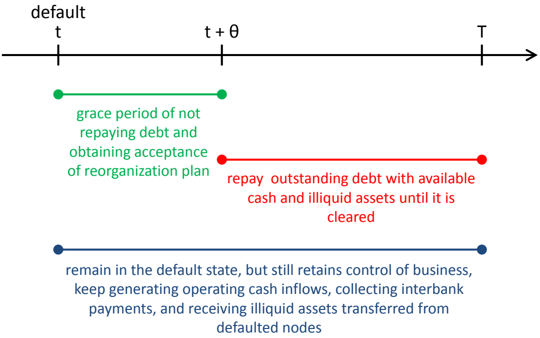
- A node i at t is said to be
  - illiquid if it cannot repay in full its liabilities, i.e. $p_i^t < l_i^t$.
  - solvent if its net asset value is positive.
  - default if it is either
    (1) illiquid and insolvent at t or
    (2) illiquid and solvent at t but not rescued by LLR.
- The net assets of node i at time t is

$$e_i^t = c_i^t + a_i^t + \underbrace{\alpha_i^t(1-\gamma_i^t)}_{asset\ recovery} - \sum_{\tau=t}^{T-1}(1+r)^{-(\tau-t)}\mathbb{E}_t\left[l_i^\tau\right],$$

where α_i^t denotes expected value of total debt owed to i at t, and γ_i^t the expected loss rate caused by nodes defaulted before t.

Default Modeling II

- Automatic stay for debtors in possession, in accordance with Chapter 11 procedures of U.S..



Unpaid Debt

The default time of node i is

$$\eta_i := \inf\left\{t : p_i^t < l_i^t\right\}.$$

Define the *unpaid debt* of node i to j as

$$W_{ij}^{t+1} = (1+r)\left[\mathbf{1}_{\eta_i = t}\left(\sum_{\tau=t}^{T-1}(1+r)^{-(\tau-t)}\mathbb{E}_t[L_{ij}^{\tau}] - \Pi_{ij}^t p_i^t\right) + \underbrace{\mathbf{1}_{t-\theta+1\le\eta_i<t}}_{within\ grace\ period} W_{ij}^t + \underbrace{\mathbf{1}_{\eta_i<t-\theta+1}}_{post-grace\ period}\left(W_{ij}^t - \frac{W_{ij}^t}{w_i^t}(c_i^t + a_i^t)\right)^+\right]$$

$$w_i^t = \sum_{j\neq i, j\in V,} W_{ij}^t$$

Asset Recovery

- The expected value of debt owed to i is computed as

$$\alpha_i^t = \sum_{j \neq i, j \in V} \left(\mathbf{1}_{\eta_j < t} W_{ji}^t + \mathbf{1}_{\eta_j \geq t} \sum_{\tau = t+1}^{T-1} (1+r)^{-(\tau - t)} \mathbb{E}_t [L_{ji}^{\tau}] \right).$$

- The expected loss rate of i at t is given by

$$\gamma_i^t = \frac{1}{\alpha_i^t} \underbrace{\left(\sum_{j \neq i, j \in V} \mathbf{1}_{\eta_j < t} (1+r)^{-(T-t)} \mathbb{E}_t \left[W_{ji}^T \right] \right)}_{\textit{unpaid debt by } T}$$

Sequence of Clearing Payments I

Building on Eisenberg & Noe (2001), define the multi-period clearing payment system, which allows specifying default events:

Definition

A sequence of $(\mathbf{p}^{t*}, \mathbf{o}^{t*})_{t=0}^{T}$ is a clearing sequence if it satisfies

- **Systemically efficient rescuing.** The LLR provides bailout loans to illiquid yet solvent nodes so to

$$\text{maximize}_{\{\mathbf{o}^t\}_{t=0}^{T-1}} \sum_{t=0}^{T-1} (1+r)^{-t} \sum_{i \in V} p_i^t.$$

Sequence of Clearing Payments II

Definition (cont.)

- **Proportional repayment of liabilities.** A node i pays $\Pi_{ij}^t p_i^{t*}$ to node $j \in V, j \neq i$ at t.
- **Absolute priority.** $p_i^t = \begin{cases} l_i^t & \text{if } \eta_i > t \\ c_i^t & \text{if } \eta_i = t \\ 0 & \text{if } \eta_i < t \end{cases}$
- **Admissible bailout.** LLR provides bailout loans only to illiquid yet solvent nodes, i.e. $o_i^t > 0 \Rightarrow c_i^t < l_i^t, e_i^t \geq 0, \eta_i > t$.
- **Just enough bailout.** Nodes are rescued with the minimum needed amount, i.e. $o_i^t > 0 \Rightarrow o_i^t = l_i^t - c_i^t$.

A Markov Decision Process (MDP) Formulation I

Define

- $X^t = (\mathbf{L}^t, \iota^t) \in \mathcal{X}$: stochastic process
- $\mathbf{o}^t \in \mathcal{O}^t$: decision process; $\mathcal{O}^t$: feasible policies.
- $s^t = (\mathbf{v}^t, \mathbf{a}^t, \mathbf{q}^t, \eta^t, \mathbf{W}^t) \in \mathcal{S}$: state at t.
- $f(s^t, \mathbf{o}^t, X^t) = s^{t+1}$: state transition function
- $\mathbb{P}^s[s^{t+1}|s^t, \mathbf{o}^t] = \mathbb{P}^X\left[\omega : s^{t+1} = f\left(s^t, \mathbf{o}^t, X^t(\omega)\right)\right]$: transition probability.

A Markov Decision Process (MDP) Formulation II

- Objective function:

$$Z^0(s^0) = \max_{\pi \in \Pi} \mathbb{E}\left[\sum_{\tau=0}^{T-1} (1+r)^{-\tau} z^\tau(s^\tau, \mathbf{o}_\pi^\tau, X^\tau)\right],$$

$$z^\tau(s^\tau, \mathbf{o}_\pi^\tau, X^\tau) := \sum_{i \in V} p_i^\tau$$

A Markov Decision Process (MDP) Formulation III

- Constraints: The bailout loan vector $\mathbf{o}_\pi^t$ must satisfy the following constraints:

$$
\begin{aligned}
\mathcal{O}^t(s^t) = \{\mathbf{o}_\pi^t \in \mathcal{O}^t : \\
& p_i^t = (1 - \mathbf{1}_{\eta_i < t}) \min\left\{l_i^t, c_i^t + o_i^t\right\}, \\
& o_i^t > 0 \Rightarrow c_i^t < l_i^t \text{ and } e_i^t \geq 0 \text{ and } \eta_i > t, \\
& o_i^t > 0 \Rightarrow o_i^t = l_i^t - c_i^t \\
& \sum_{i \in V} (q_i^t + o_i^t) \leq B(1 + r_c)^t\}
\end{aligned}
$$

Policy Computation

- For high dimensional financial networks, MDP becomes computational intractable.
- Develop a suboptimal approach:
  - Choose heuristic bailout allocation rules
  - Approximate the objective value $Z^0(s^0)$ of each heuristics via Monte-Carlo simulations
  - Suitably combine the heuristics so to select the best suboptimal policy in each decision epoch
- Analyze behavior of heuristic algorithms in this talk (see paper for suboptimal strategies)

Myopic Heuristic

- Computes solution maximizing the single period payment function

$$\mathbf{o}^t_{\pi_1} = \arg\max_{\mathbf{o}^t \in \mathcal{O}^t} z^t(s^t, \mathbf{o}^t, X^t) \qquad z^t_1 = \sum_{i \in V} p^t_i(s^t, \mathbf{o}^t_{\pi_1}, X^t)$$

Pre-allocation Heuristic

(1) Computes the ratio of unpaid debt caused by defaults in decision epoch t, i.e.

$$R^t = \frac{\sum_{i\in V} \mathbf{1}_{\eta_i=t} w_i^T (1+r)^{-T}}{\sum_{\tau=0}^{T-1} \sum_{i\in V} \mathbf{1}_{\eta_i=\tau} w_i^T (1+r)^{-T}}$$

(2) Preallocate budget $B^t = B \times R^t$ to decision epoch t.

(3) Compute

$$\mathbf{o}_{\pi_2}^t = \arg\max_{\mathbf{o}^t \in \mathcal{O}^t} z^t(s^t, \mathbf{o}^t, X^t) \qquad z_2^t = \sum_{i\in V} p_i^t(s^t, \mathbf{o}_{\pi_2}^t, X^t)$$

Systemic Risk Measures

For $Y \subseteq V$, define

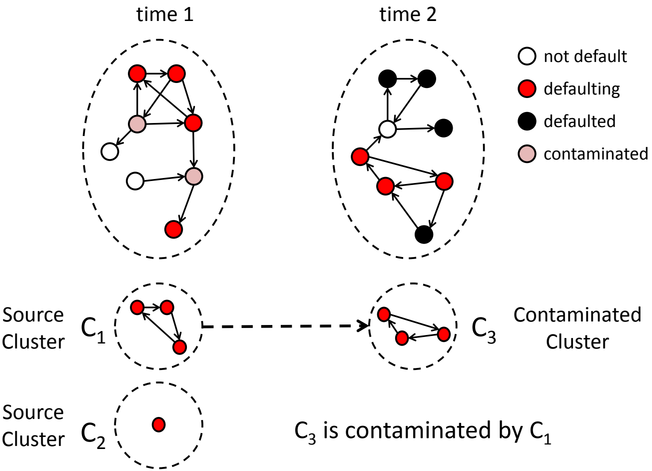
$$UL_Y = \frac{\sum_{t=0}^{T-1}(1+r)^{-t}\sum_{i\in Y}(l_i^t - p_i^t(s^t, \mathbf{o}^t, X^t))}{\sum_{t=0}^{T-1}(1+r)^{-t}\sum_{i\in V} l_i^t},$$

and residual systemic risk generated by Y as,

$$RS_Y = \mathbb{E}\left[UL_Y\right],$$

i.e. the percentage of liabilities unpaid by the nodes in Y, after accounting for optimal bailout policy.

Systemic Graph



C_3 is contaminated by C_1

Systemic Risk Allocation

Let S and C denote respectively the set of nodes in source and contaminated clusters. The residual systemic risk attributed to time t is

$$RS_V^t = \mathbb{E}\left[R^t UL_V\right],$$

The amount generated by source clusters is

$$RS_S^t = \mathbb{E}\left[R^t UL_S\right],$$

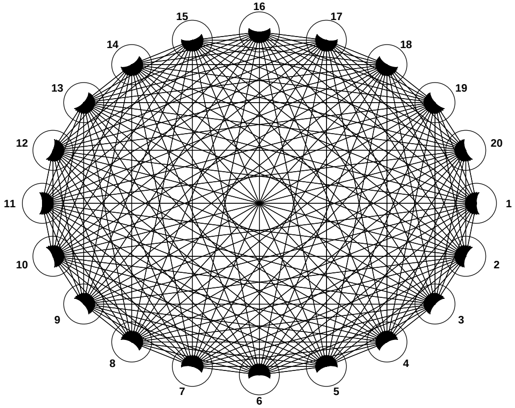
while the amount due to contaminated clusters is

$$RS_C^t = \mathbb{E}\left[R^t UL_C\right].$$

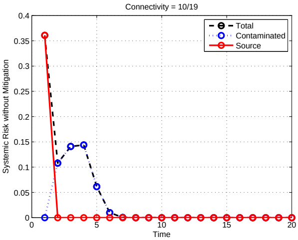
Experiment Setting

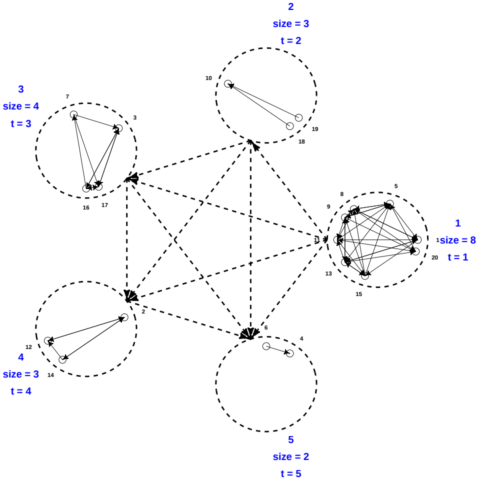
- Consider two networks, homogeneous and heterogeneous, with $n = 20$, $T = 20$, and $\Delta t = 1$
- Homogeneous network: liabilities and operating cash inflows are i.i.d. Gaussian
- Heterogeneous network: liabilities and operating cash inflows are independently distributed Gaussian

Homogeneous Network

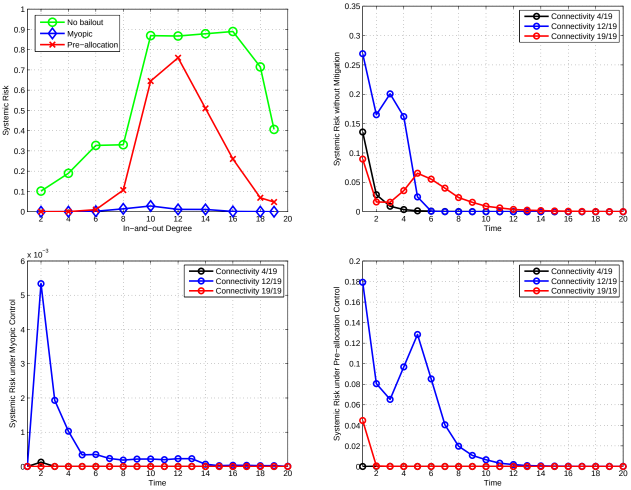


Homogeneous Network: Systemic Risk I

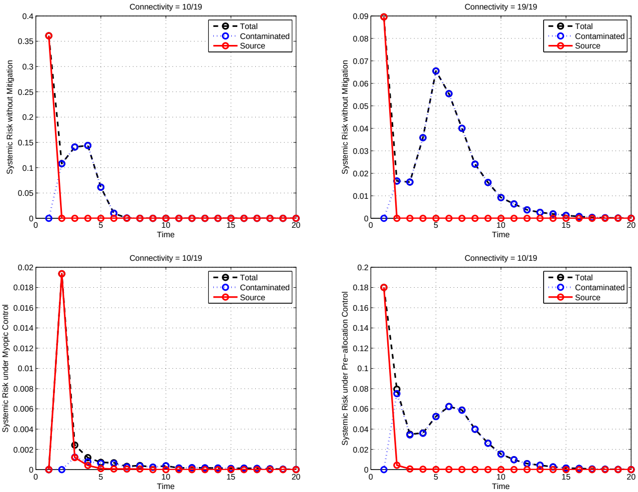




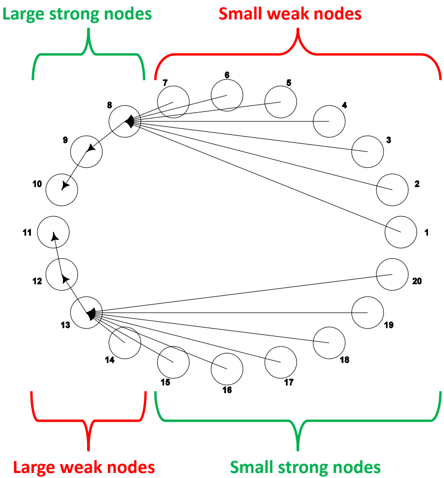
Homogeneous Network: Systemic Risk II



Homogeneous Network: Systemic Risk III

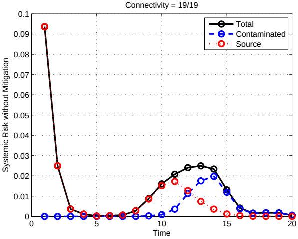


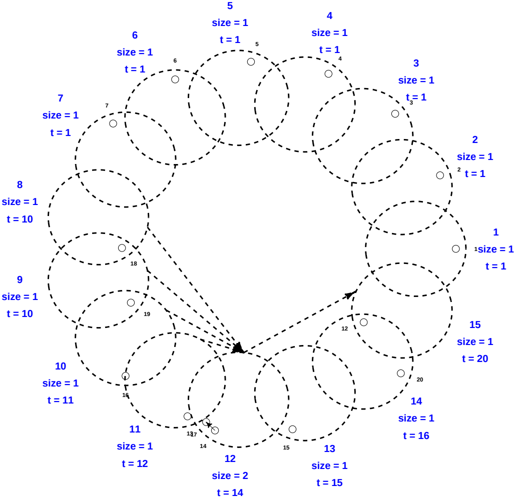
Heterogeneous Network



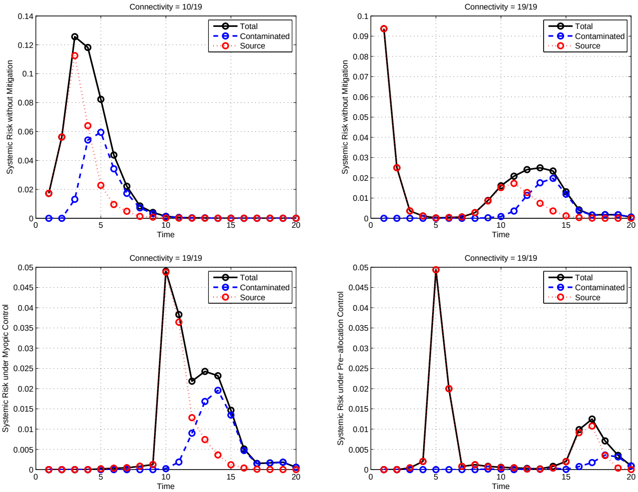
- Operating cash inflow < liabilities
- Small/large: low/high balance sheet size
- Weak/strong: low/high initial available cash

Heterogeneous Network: Systemic Risk I





Heterogeneous Network: Systemic Risk II



Conclusion

- Developed a multi-period framework to quantify systemic risk propagation and mitigation effects
- Clearing payments and bailout strategies recovered as the solution of Markov decision process
- Homogeneous network: systemic risk has an inverted U shape, and can be significantly reduced using myopic strategies.
- Heterogeneous network: systemically important nodes may change over time depending on the state of network.